Ethno Med, 11(2): 146-157 (2017)
DOI: 10.31901/24566772.2017/11.02.02

Medical Assessment Based on Generalized Gamma Distribution Generalized Linear Mixed Models

Hong Li , Yuantao Xie[2] and Juan Yang[3]

[1]School of Economics, Peking University, China
[2]School of Insurance and Economics, University of International Business and Economics, Beijing, 100029 China
[3]Institute of Comprehensive Development, Chinese Academy of Science and Technology for Development, Beijing 100038, China
Telephone: +86 13718556415; +86 13718189492
E-mail: <xieyuantao@gmail.com> , <yangj@casted.org.cn>

KEYWORDS Iterated Weighted Least Squares. Penalized Quasi-likelihood. Recovery Curve. Pseudo-likelihood. Shrinkage Estimate

ABSTRACT This paper proposes an analysis model for a Generalized Gamma distribution, data is distributed based on the Generalized Linear Mixed Models. The authors estimate the parameters, construct the score test as well as the shrinkage estimation, and explore how to deal with the medical service assessment. The recovery curves are obtained with dynamic investigation data from the US Shriners Hospitals and achieved medical service assessment in the empirical analysis. The recovery curve can explore the recovery of the burned children visually and comprehensively. With this model it is easy to give a comprehensive assessment on burn recovery from different aspects such as physics, social psychology, and so on. The method proposed in this paper is also available in the comprehensive assessment which is generally used in a social research.

INTRODUCTION

With the improvement of medical technology and the development of science and technology, many medical institutions around the world have begun to work on the assessment of the medical services. For most cases, the criteria for medical assessment are not limited to the cure or the survival rates, and the criteria used are multi-dimensional, including physical, social, psychological ones. Regarding the multidimensional index modeling, the possible problems are: the data are often derived from the scale, it is not likely to show a normal distribution of the law; it is difficult to reflect the social psychological characteristics of the patients; medical service assessment results from some traditional methods such as AHP, Delphi and Fuzzy comprehensive assessment, are often a simple score, which doesn't help the dynamic development process.

The framework of a generalized linear mixed model is explored in this research area. The researchers constructed an analysis model for the data that follows the Gamma distribution to display the dynamic development process of recovery and to evaluate the medical services. In the empirical analysis, the comprehensive rehabilitation status of the patient's social psychology and other aspects were analyzed by using the dynamic tracking survey scale (American ABA Association Table) of the children's rehabilitation status of the Shriners Hospitals.

Review

For the medical assessment data, it usually does not follow a normal distribution and the classical regression analysis technique is invalid. Scholars attempted to improve the analysis from two aspects.

First, assume that the data satisfies the exponential distribution family (including exponential, Poisson, binomial, negative binomial, multinomial, single parameter Gamma, Gauss and inverse Gauss distribution), Nelder and Wedderburn (1972) developed a Generalized Linear Model. The random effects (Zeger et al. 1988) and the repeated observations are often used in the biomedical data. For parameter estimation, Breslow and Clayton (1993) proposed the Quasi-Likelihood and the marginal Quasi-Likelihood method, Wolfinger and Connell 'O (1993) proposed the Pseudo-likelihood method. Gurka et al. (2006) introduced the Box-Cox transformation to a linear mixed model, see also Jiang (2007) for the specific model. Antonio and Beirlant (2007) introduced the actuarial application with GLMMs (Generalized Linear Mixed Models). Villemereuil et al. (2016) discussed the modeling of quantitative genetic parameters for non-normal traits, showed that it's easy to incorporate the models into predicting evolutionary trajectories. O'Hara (2016) showed that it's easy to incorporate the models into predicting evolutionary trajectories to analyze the majority of ecological data. Chen and Wehrly (2016) expanded the GLMMs for jointly modeling the clustered mixed outcomes. Hoyer and Kuss (2016) compared different diagnostic tests according to GLMMs. Most research on random effect is based on the hypothesis that it follows a normal distribution. Drikvandi et al. (2016) expanded the hypothesis by way of gradient function proposed by Verbeke and Molenberghs (2013), they used bootstrap algorithm for simulation and testing. Diaz (2016) used GLMMs to measure the individual benefit of medical treatment.

Later, the model is established to a more general distribution, such as the Gamma distribution (Clayton and Cuzick 1985), and the inverse Gauss distribution (Hougaard 1986). Both studies have indicated that the reliability, the generalized survival time (vehicle's traveling total mileage, medical expenses and medical equipment use frequency), and the follow-up data often satisfy the generalized Gamma distribution (Lawless 2003). Keiding et al. (1997) pointed out that the conditional covariates are sensitive to the error of the random effects distribution. Kwong et al. (2003) further analyzed the risk of the model error. Xie and Yang (2010) constructed the Generalized linear mixed model of the generalized Gamma distribution to reduce the risk of the model error, and can make a full use of the dependence structure between the response variables, which have a very good flexibility; Xie et al. (2012) constructed the reliability pricing model based on the generalized linear mixed model, but that the previous research has not given a data-based empirical analysis to illustrate the results. As referring to Gamma distributions, Mcmahon (2011) employed an empirical analysis based on Generalized linear mixed model and calibration for gamma random variables. This paper studied the robustness via simulation. Chen and Huang (2014) modeled the drug concentration-time profiles by way of gamma mixed-effect model. Baldi et al. (2010) built the empirical assessment model for outcomes in oral health research by way of gamma GLMMs.

Innovation

Traditional rehabilitation analysis mainly uses the expert scoring method to analyze the improvement of the pathological condition and gives the total score to evaluate the effect, which is limited to the medical research. In this paper, the researchers take into account the social psychology and other aspects, which gives a comprehensive measure of the rehabilitation status. In this study, the researchers establish the generalized Gamma distribution model, based on the twelve dimensional variables of the children's rehabilitation, to draw the rehabilitation curve, and analyze the dynamic assessment of children's physical and mental status.

METHODOLOGY

Specification

For this study, the physical and the mental rehabilitation time of the children themselves can be seen as the survival time, which can be considered as the generalized Gamma distribution. The generalized Gamma distribution model of the generalized linear mixed model is described as follows:

$$Q(\mu_i, y_i) = \frac{y_i\theta_i - b(\theta_i)}{a(\phi_i)}$$

$$\eta = X\beta + Zu \tag{1}$$

$$y^* = \mu + e = E\,(y^* \mid i) + e \tag{2}$$

η represents the linear predictor $g(\mu)$ represents the link function μ represents the conditional means. X represents the design matrix for the fixed effect β represents the fixed effect z represents the design matrix for the random effect, represents the random effect μ following a MVN (0, G) distribution c represents is disturbance R is the covariate matrix, which is the variance matrix for the random effect G and repeated measure matrix R which contains unknown parameters ϕ where,

$$E\begin{bmatrix}u\\e\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} \quad Var\begin{bmatrix}u\\e\end{bmatrix} = \begin{bmatrix}G & 0\\0 & R\end{bmatrix} \tag{3}$$

Suppose, the medical service assessment follows a generalized Gamma distribution (Mohani et al. 2014), then the log-likelihood function can be expressed as:

(4)

where, $y^* = y^\tau$, $\theta = \frac{1}{\mu}$, $\phi = \frac{1}{v}$, $b(\theta) = \theta$, $a(\phi) = -\phi$ (Xie and Yang 2010).

$$E(y|u) = h\,(X\beta + Zu) \tag{5}$$

$$Var(y|u) = R = Var(e) \tag{6}$$

Estimation

The traditional generalized linear mixed model analysis is based on the Penalized Quasi-Likelihood (PQL) method or the Marginal Quasi-Likelihood (MQL) method. In the former methods, the full likelihood function is approximated by the Laplace method. Differentiating the scoring equation with respect to the parameter ४, the maximum likelihood estimator is obtained. The latter is also used to analyze the variables of the normal distribution. The Quasi-Likelihood method is simple and direct, and the Marginal Quasi Likelihood method is stable (Ahmad and Adil 2014). When the first order moment E(y)=μ is set correctly, even if the variance components are set, the generalized estimating equation can still provide a consistent estimate of the fixed effect.

Breslow and Clayton (1993) introduced the Quasi-Likelihood method for parameter estimation, and compared the two different Quasi-Likelihood methods, that transform the solution of the scoring system into the iterative (RE) weighted least squares (Nelder and Wedderburn 1972). Vonesh et al. (2000) further improved the procedure by introducing the first or second order conditional moments in the estimation equation. Quasi-Likelihood method can be considered as a special case of the Pseudo-Likelihood method proposed by Wolfinger and Connel (1993). In practical application, the latter is simpler to implement.

After introducing the generalized Gamma distribution parameters, the parameter estimation becomes very complicated, and the nonlinear search algorithm is proposed in this paper.

Suppose τ is known, it is easy to estimate other unknown parameters with the theory support of the Generalized Linear Mixed Models; some other models such as random intercept models, variable-coefficient models, repeated models, and spatial models can be easily derived from the GGDGLMM.

The Laplace's method or the Taylor series approximation is generally used to get the linear approximation of the likelihood function. Partial derivative technology can be used to build score functions. Then the parameters estimation can be derived from iteration estimation.

Applying Laplace's method, the approximation of the Marginal Quasi-Likelihood (Goldstein 1991) eventually leads to the estimation function based on the Penalized Quasi-Likelihood (PQL) (Green 1987) for the mean parameters and the Quasi-Likelihood for the variance-covariance parameters (Breslow and Clayton 1993). The research of Wolfinger and O'Connell (1993) showed that both PQL and MQL are special cases of the Pseudo-Likelihood method (Carroll and Ruppert 1988). In this paper, the Pseudo-Likelihood method is utilized to reach the joint likelihood function.

Score Functions: Step 1, let $\hat{\beta}$ and $\hat{\mu}$ be the known estimates of β and μ, then $\hat{\mu} = g^{-1}(X\hat{\beta} + Z\hat{u})$

The researchers can get the Taylor series approximation to e

$$\tilde{e} = y^* - \hat{\mu} - (g^{-1})'(X\hat{\beta} + Z\hat{u})(X\beta - X\hat{\beta} + Zu - Z\hat{u}) \tag{8}$$

Step 2, according to Laird and Louis (1982) and Lindstrom and Bates (1990), the conditional distribution $e|\beta, \mu$ could be approximated by a Gaussian Distribution, that is, $\tilde{e} \mid \beta, u \sim N(0, R_\mu^{1/2} A R_\mu^{1/2})$

$$y^{**} \mid \beta, u \sim N\left[X\beta\ \beta\ Zu, g'(\hat{\mu}) R_{\mu}^{1/2} A R_{\mu}^{1/2} g'(\hat{\mu})\right]$$

Step 3, substitute the μ with μ in variance-covariance matrix

$$g'(\hat{\mu})(y^* - \hat{\mu}) \mid \beta, u \sim N\left[X\beta - X\hat{\beta} + Zu - Z\hat{u}, g'(\hat{\mu}) R_{\hat{\mu}}^{1/2} A R_{\hat{\mu}}^{1/2} g'(\hat{\mu})\right] \tag{9}$$

Where $g'(\hat{\mu})$ is a diagonal matrix with the elements

$$g'(\hat{\mu}_i) = \left[(g^{-1})'(X_i\hat{\beta} + Z_i\hat{u})\right]^{-1} \tag{10}$$

Define,

$$y^{**} = \hat{\eta} + (y^* - \hat{\mu})\, D^{-1} = g(\hat{\mu}) + g'(\hat{\mu})(y^* - \hat{\mu})$$

Where, $D = \left[\frac{\partial \mu}{\partial \eta}\right]$ then the researchers approximate the conditional distribution with a Gaussian distribution

(11)

Define.

$$\hat{W} = \hat{D}^T \hat{R}^{-2} \hat{D} = R^{-2}_{\mu} [g'(\hat{\mu})]^{-2} \tag{12}$$

Where, $D = \left[\frac{\partial \mu}{\partial \eta}\right]$ For the canonical link functions $\hat{W} = R_\mu^{-1}$.

In fact, (y^{**}) is the Taylor series approximation to (y^*), as well as the approximation to the response in the Iterated Weighted Least Squares (IWLS) (Nelder and Wedderburn 1972) problem.

Fisher Scoring Methods: The researchers got the iterated estimates of β and μ from the following fisher score functions:

$$\begin{bmatrix} X^TWX & X^TWZ \\ Z^TWX & Z^TWZ+G^{-1} \end{bmatrix} \begin{bmatrix} \beta \\ u \end{bmatrix} = \begin{bmatrix} X^TWy^{**} \\ Z^TWy^{**} \end{bmatrix} \quad (13)$$

Where, $W = diag\left\{\{b''(\theta_i)a(\phi_i)[g'(\mu_i)]^2\}^{-1}\right\}$ is the iterated weighted diagonal matrix. $y^{**}=\hat{\eta}+(y^*-\hat{\mu})D^{-1}$ is the working matrix. $\mathbf{D} = \left[\frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\eta}}\right]$, and R is the variance matrix. Both D and R are unknown, so they are replaced by their estimates.

Iterated Weighted Least Squares (IWLS): Iterated Weighted Least Squares (IWLS) are generally used to solve the score functions.

(1) Non-singular Variance Matrix

Let $V^{-1} = \left(\Delta V_0 \Delta + ZGZ^T\right)^{-1} = \left(W^{-1} + ZGZ^T\right)^{-1}$, the researchers can get the estimate for the fixed effects $\hat{\beta}$ first in each iteration

$$\hat{\beta}=(X^TV^{-1}X)^{-1}X^TV^{-1}y^{**} \quad (14)$$

Then get the shrinkage estimate (Robinson 1991) of the random effects

$$\hat{u} = GZ^TV^{-1}\left(y^{**} - X\hat{\beta}\right) \quad (15)$$

(2) Singular Variance Matrix

Recall that $\hat{G}$ should be a non-singular matrix in the IWLS estimation, however, some elements of the matrix equal 0 during the iteration. The researchers used the Cholesky method to modify the matrix (Henderson, 1984).

Suppose $\hat{L}$ is the Cholesky lower triangular root of $\hat{G}$ such that $\hat{G} = \hat{L}\hat{L}^T$. Suppose $\hat{u} = \hat{L}\hat{v}$, score functions can be transformed into the following:

$$\begin{bmatrix} X^T\hat{W}X & X^T\hat{W}Z\hat{L} \\ \hat{L}^TZ^T\hat{W}X & \hat{L}^TZ^T\hat{W}Z\hat{L}+I \end{bmatrix} \begin{bmatrix} \beta \\ v \end{bmatrix} = \begin{bmatrix} X^T\hat{W}y^{**} \\ \hat{L}^TZ^T\hat{W}y^{**} \end{bmatrix} \quad (16)$$

β is an unbiased and consistent estimate of β if the conditional means are specified correctly. β possesses an approximating Gaussian distribution (McCullagh and Nelder 1989). The corresponding first order variance approximation is the following:

$$Cov(\hat{\beta})=(X^TV^{-1}X)^{-1} \quad (17)$$

Estimation of Distribution Parameters: Replacing all the parameters with their estimates, the researchers get an approximate profile quasi-likelihood function of φ. $\hat{\varphi}$ could be estimated utilizing the partial derivation technology

$$\hat{\varphi} = \frac{(y''-X\hat{\beta})'V\text{-}1(y''-X\hat{\beta})}{n-p} \quad (18)$$

The Pseudo-Likelihood(PL) method or Restricted Pseudo-Likelihood (REPL) could be used to derive PL estimate or REPL estimate of ϕ. PQL method can be seen as a special case of $\phi=1$.

The Algorithm for GGDGLMMs: All the estimation in section 2 is under the assumption that τ is given, all the estimated parameters can be seen as the function of τ, such as $\hat{\delta}(\tau)$, $\hat{u}(\tau)$, and $\hat{v}(\tau)$. The estimate of the joint likelihood function is $L\left(\hat{\delta}(\tau), \hat{\mu}(\tau), \hat{v}(\tau), \tau\right)$.

$y^*=y^{\tau}$ is a monotonic function when $\tau>0$. The estimation of τ is the same thing as solving the following problem:

$$\underset{\tau}{Max}: L(\tau) = L\left(\hat{\delta}(\tau), \hat{\mu}(\tau), \hat{v}(\tau), \tau\right)$$

It is very difficult to get the algebraic solution. A non-symmetry dichotomic search method might be proper for this problem for the sake of $\tau>0$. The algorithm can be shown in the following graph where, d is a step parameter and is an adjustment parameter that adjust the step length. The coding design is seen in Figure 1.

This algorithm can be easily achieved by way of using the SAS GENMOD module repeatedly. Generally speaking, it will converge after 10 loops (Table 1).

Shrinkage Estimate and Score Test

According to the descriptive statistics and the hypothesis testing, the researchers know that the medical assessment scores follows a generalized Gamma distribution, but cannot determine which specific distribution of the generalized Gamma distributions is proper, Gamma, Weibull or exponential distribution. The simpler, the better. The fewer parameters, the better. Therefore, we consider the score test for the shrinkage estimation (Xie and Yang 2015).

The conditional probability density function can be expressed as:

$$f(y \mid X, Z) = \tau\left(ve^{X\beta}e^{Zu}\right)^v \cdot y^{\tau v-1} \cdot \exp\left\{-ve^{X\beta}e^{Zu}y^{\tau}\right\}/\Gamma(v) \quad (19)$$

Applying the Taylor Expansion for (19) at $e^{Zu}=1$ the researchers obtain the approximation:

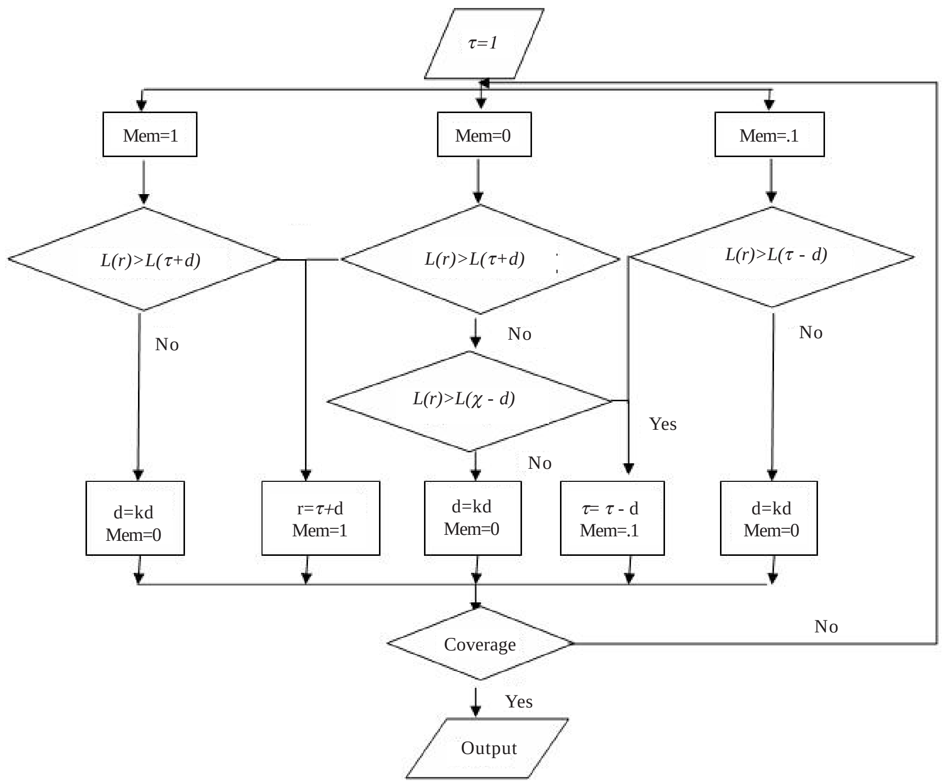

Fig. 1. The algorithm for GGDGLMMs
Source: Authors

$$f(y\mid X,Z)\approx f\left(y\mid X,e^{Zu}=1\right)\Big\{1+\left(e^{Zu}-1\right)\left(v-ve^{X\beta}y^{\tau}\right)+\frac{1}{2}\left(e^{Zu}-1\right)^{2}\left[\left(v-ve^{X\beta}y^{\tau}\right)^{2}-v\right]\Big\} \qquad (20)$$

Integrating about e^{Za}

$$f(y\mid X)=E_{e^{Zu}}\left(f(y\mid X,Z)\right)\approx f\left(y\mid X,e^{Zu}=1\right)\Big\{1+\frac{1}{2}Var\left(e^{Zu}\right)\left[\left(v-ve^{X\beta}y^{\tau}\right)^{2}-v\right]\Big\} \qquad (21)$$

The Log-Likelihood function is

$$L\approx\sum\left\{\log\tau+v\log\left(ve^{X\beta}\right)+(\tau v-1)\log y-ve^{X\beta}y^{\tau}-\log\Gamma(v)\right\}+\sum\log\Big\{1+\frac{1}{2}\sigma^{2}\left[\left(v-ve^{X\beta}y^{\tau}\right)^{2}-v\right]\Big\} \qquad (22)$$

The score test can be set. The hypothesis test for the generalized linear mixed model when the generalized Gamma distribution is

$$H_0:\quad \sigma^2=0,\ \tau=1$$

The unknown parameters are denoted as δ which can be spited as two parts $\delta=\left(\delta_1^T\ \delta_2^T\right)^T$, where $\delta_1^T=\left(\sigma^2\ \tau\right)$, δ_2^T represent other parameters. The Score function is denoted as $s(\delta_0)$ and the score vector containing the first parameter vector is denoted as $s_1(\delta_0)$, $\left.\frac{\partial L}{\partial\sigma^2}\right|_{H_0}=s_{11}(\delta_0)$. The sub block matrix of the inverse matrix of the corresponding information matrix is $M_{11}(\delta_0)$. The researchers construct the joint score test under the original hypothesis.

$$LM_{hm}=s_1^T\left(\hat{\delta}_0\right)\cdot M_{11}\left(\hat{\delta}_0\right)\cdot s_1\left(\hat{\delta}_0\right)\sim\chi^2(2) \qquad (23)$$

where $\hat{\delta}_0$ represents the estimate for $\hat{\delta}_0$. The score test for the Weibull distribution and for the exponential distribution are similar. For the medical service assessment, the researchers establish the generalized Gamma distribution model based on the original data, employ the shrinkage test based on the score test, select the general-

Table 1: Algorithm and coding

1	*Initialize* $\hat{\tau}=1$, d, k, *mem=0 and calculate* y^*;
2	Do until ($d < 0.001$ or i<N);
3	Initial $\hat{\mu}^{(new)}$、$\hat{\beta}^{(new)}$、$\hat{u}^{(new)}$、$\hat{\varphi}^{(new)}$;
4	Do until((| $\hat{\beta}^{(new)}$ - $\hat{\beta}$ |<1e-5 and | $\hat{u}^{(new)}$ - $\hat{u}$ |<1e-5 and | $\hat{\varphi}^{new}$ - $\hat{\varphi}$ |<1e-5) or j<N);
5	$\hat{\mu}=\hat{\mu}^{(new)}$, $\hat{\beta}=\hat{\beta}^{(new)}$, $\hat{u}=\hat{u}^{(new)}$, $\hat{\varphi}=\hat{\varphi}^{(new)}$;
6	Calculate y^{**};
7	Get $\hat{D}$、$\hat{R}$ from $\hat{\varphi}$, then calculate $\hat{W}$;
8	Estimate $\hat{\varphi}^{new}$ by way of Ml, or estimate $\hat{\varphi}^{new}$ by way of REML;
9	Estimate $\hat{\beta}^{(new)}$、$\hat{u}^{(new)}$;
10	Estimate $\hat{\mu}^{(new)}$;
11	End;
12	Calculate SS (τ);
13	If mem=1 then go to 15;
14	If mem=-1 then go to 16;
15	Loop a round 3 -12 to calculate SS ($\tau+d$); if SS ($\tau+d$) < SS (τ); then $\tau=\tau+d$ mem=1; go to 18;
16	Loop around 3 -12 to calculate SS ($\tau-d$); if SS ($\tau-d$) < SS (τ); then $\tau=\tau-d$ mem=-1; go to 18;
17	$d=kd$; mem=0;
18	$i=i+1$;
19	End;
20	Output;

ized linear mixed model, and set up the forecast model.

RESULTS

The American Shriners Hospitals are professional treatment hospitals of burns and diseases. These hospitals are located in all states of the United States, thus forming the first hierarchical data. The Hospitals schedule the burn patients for a follow-up investigation, where each patient has a return visit every 3 months, 6 months, 12 months, 18 months, 24 months and 36 months after being discharged.

The questionnaire used by the American Burn Association (ABA) unified the formulation of the table, and was in accordance with the recommendations of sociology experts, medical experts, and psychologists. The 12 dimensions reflect the physiological, psychological, social and other aspects.

Taking into account the social psychological assessment has many subjective factors, combined with the diversity of individual patients of the disease, the diversity of the social psychology and sample of different patients, the data is more complex. We need to consider the fixed effects, the random effects and the repeated observations. The researchers collected the children's assessment on their own, as well as the assessment of the children's parents, to get the corresponding survey data, and to calculate the corresponding dimension value.

Using the repeated observations, the reliability level of the 12 dimensions in the 7 dimensions was measured by the Intragroup Consistent ICC (Coefficient). The researchers selected the children aged 5-18 and analyzed the data from two aspects, which are the children's own assessment and the assessment of the children's parents.

Statistics

The total sample size was 158, and the average burnt area was 28.6 percent. 53 percent of burn children was white. 87 percent is male. Spanish speaking children accounted for 37 percent. The descriptive statistics is given in Table 2.

Table 2: Descriptive statistics

Variable	*No.*	*Mean*	*Std*
Male	158	0.87	0.34
White	158	0.53	0.5
Tbsa_burned (%)	158	28.6	21.9
Tbsa_burn20	158	0.57	0.5
Spanish1	158	0.37	0.49

A simple descriptive statistic of all the 12 dimensions was calculated from two angles of the children and the parents (Table 3).

Based on the comparison results, the parents and the children responded quite differently in the assessment of the rehabilitation status. Some of the factors were statistically significant, such as the upperfx, itch, transmob, and appear. Along with the general description, the researchers also present the recovery curves for both the parents and the children's assessment.

Fitting Result

The data are fitted to the generalized Gamma distribution. The selected covariates included TBSA, time, and factors including age, gender, race, language, and the interaction effect of "whether or not to hurt the hands and whether or not to hurt the face." For both the parents and the children, the model results are given in Table 4 (only the distribution parameters).

Table 4: Distribution parameter estimate

	τ (parents)	*τ (parents)*	*τ (children)*	*τ (parents)*
Upperfx	0.3705	1.267	0.3231	1.012
Physport	0.5759	1.451	0.5822	1.236
Transmob	0.3667	0.912	0.3296	1.102
Pain	0.3884	1.249	0.4265	1.210
Itch	0.5568	1.127	0.5955	0.906
Appear	0.6239	0.924	0.4476	0.978
Comply	0.2705	1.079		
Satisfy	0.3470	0.891	0.3412	0.912
Emotions	0.3297	0.974	0.2341	0.939
Family			0.5096	1.067
Concernp	0.9021	0.874	1.1601	0.892
School	0.2782	0.976	0.3123	0.966

The space section indicates that there are no convergence criteria. The researchers establish an

Table 3: Rehabilitation status of children with burn injury (comparison between children and their parents)

Dimension	*Children*			*Parents*			*Comparison*	
	No.	*Mean*	*Std*	*No.*	*Mean*	*Std*	*diff*	*Pr > |t|*
Upperfx	157	83.77	22.12	157	85.78	19.51	1.34	0.0478
Physport	154	67.61	30.96	157	69.07	31.07	0.44	0.3286
Transmob	157	83.41	23.38	157	86.16	21.56	1.65	0.004
Pain	156	72.16	22.98	157	73.85	24.3	0.79	0.2802
Itch	157	59.19	25.88	157	55.21	28.33	-1.54	0.015
Appear	156	57.96	27.74	158	61.51	24.27	1.29	0.0319
Comply	152	91.09	14.18	155	92.49	11.31	0.62	0.289
Satisfy	156	72.5	20.95	155	73.71	19.97	0.76	0.3082
Emotions	157	82.54	17.46	158	83.76	16.77	0.69	0.3257
Family	154	66.68	25.74	157	67.53	24.85	0.67	0.3932
Concernp	151	32.51	25.39	158	29.3	28.47	-1.05	0.0831
School	95	58.68	17.57	100	61.42	18.11	1.69	0.0705

exponential distribution model to fit the curve of recovery.

Score Test

The generalized Gamma distribution includes Gamma distribution, Weibull distribution and the exponential distribution as special cases, which is difficult to distinguish in empirical analysis. The results of the joint score test for different distributions are presented separately in Table 5 [Joint score test results for gamma distribution assumption and Weibull distribution assumption are given in Table 5 respectively.]

The critical value for the corresponding Chi-square distribution under the 0.05 significant level is 5.9915. The transmob dimension, itch dimension, appear dimension and satisfy dimension for parents cannot reject the hypothesis of the Gamma distribution. The emotions dimension for parents cannot reject the hypothesis of the Weibull distribution. The transmob, itch, satisfy, appear and family dimension for children cannot reject the hypothesis of the Gamma distribution. The pain, emotions and family dimension for children cannot reject the hypothesis of the Weibull distribution. Other tests were rejected.

It is worth mentioning that, neither the Gamma distribution nor the Weibull distribution of the original hypothesis can be rejected in the family dimension for children. Both the Gamma distribution and the Weibull distribution are rejected in the comply dimension and concern dimension for parents.

Rehabilitation Curve

The aim is to explain and describe the effect of the health care for the purpose of establishing a generalized linear mixed model of the generalized Gamma distribution. The disease recovery curve can be well realized in this assessment.

Using the predicted values of the hybrid model, the recovery curves are as follows (in the back of the graph, the blue curve indicates the child, the red curve indicates the parent):

Obviously, in the appear dimension, the children's result is higher compared to their parents. The results increase rapidly in beginning and gradually tends to be stable. In the comply (complain) dimension, the values from the children were high at the beginning then decreased rapidly. In contrast, the values from the parents do not decrease much. It seems that after discharge, the children gave high volumes of complaints but then decreased rapidly during the rehabilitation process, where as parents did not share the same experience. In the emotion and family dimension, the recovery curves of the children and the parents have consistent shapes, and eventually tends to be the same. It is more objective to record and recognize this phenomenon. For the itch dimension of the parents and the children, the recovery curve almost overlaps. The conclusion of the pain dimension was similar to the previous.

In the physport dimension, the value of the parent's dimension was consistent with their child's. The dimension is relatively objective, and therefore it is easy to obtain the consistent result between parents and the child's assessment. Looking at the satisfaction on the current status, both the parents and the children have their satisfaction level gradually increased and stabilized. This is the unique special recovery curve. The school dimension almost shows no change. Lastly, for the upperf (upper limb function) dimension, the children have a higher assessment than

Table 5: Joint score test

	Gamma (parents)	*Weibull (parents)*	*Gamma (children)*	*Weibull (children)*
Upperfx	6.2950	12.3135	6.3456	21.3098
Physport	9.5695	12.5295	9.5552	9.5650
Transmob	3.3594	8.3431	4.3314	8.3609
Pain	12.4245	9.4188	15.4374	5.4285
Itch	4.6208	12.6178	3.6336	12.5978
Appear	5.4571	16.4737	2.5260	12.5171
Comply	6.1696	6.1439		
Satisfy	5.3395	9.3590	5.3227	8.3773
Emotions	11.2698	4.2628	10.3200	5.3183
Family			4.4279	4.4427
Concernp	21.1542	11.1720	11.1318	13.0988
School	9.2126	10.2014	7.3465	8.8215

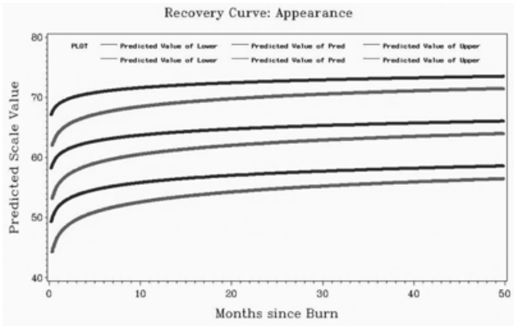

Fig. 1. Appearance

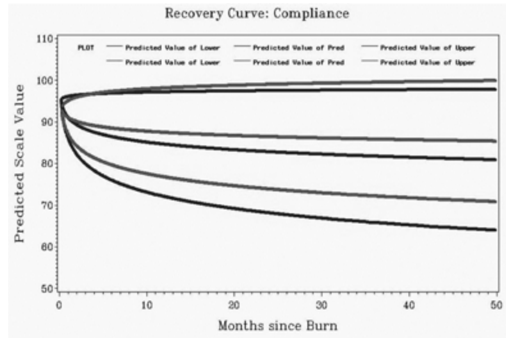

Fig. 2. Compliance

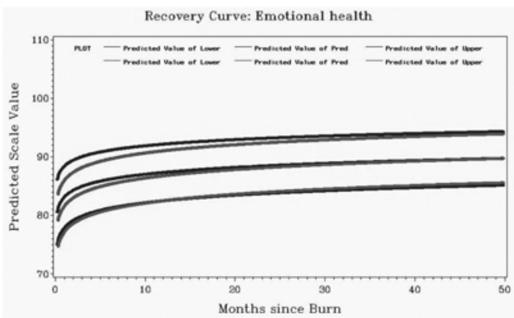

Fig. 3. Emotional health

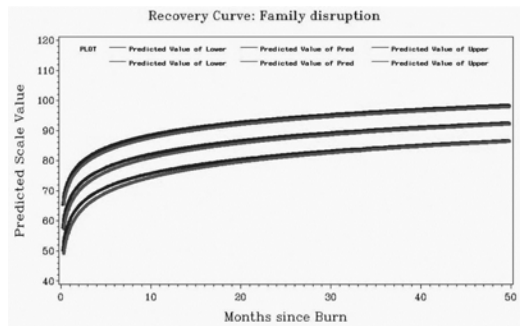

Fig. 4. Family disruption

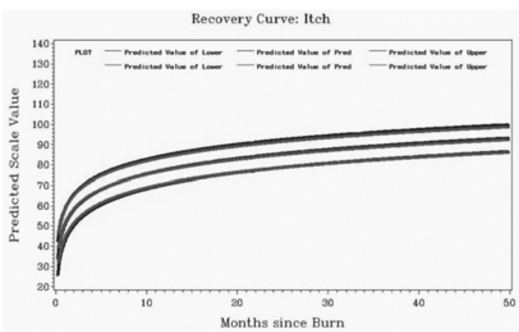

Fig. 5. Itch

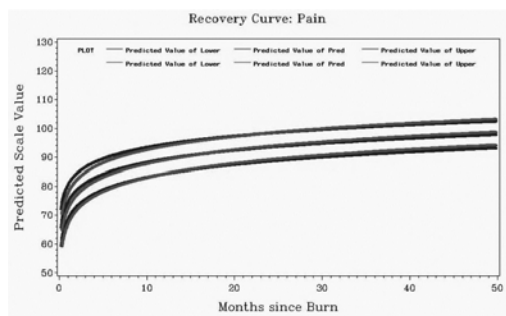

Fig. 6. Pain

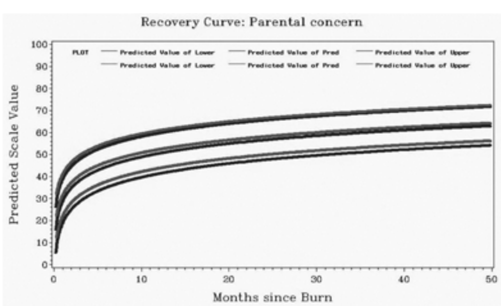

Fig. 7. Parental concern

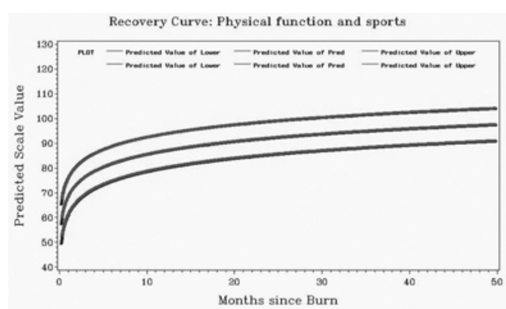

Fig. 8. Physical function and sport

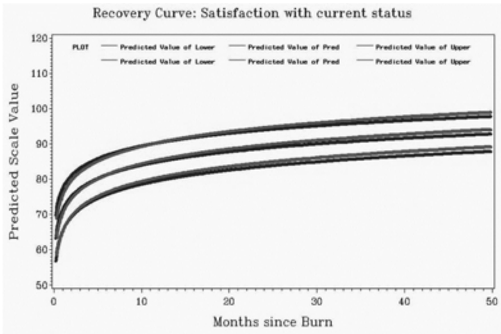

Fig. 9. Satisfaction

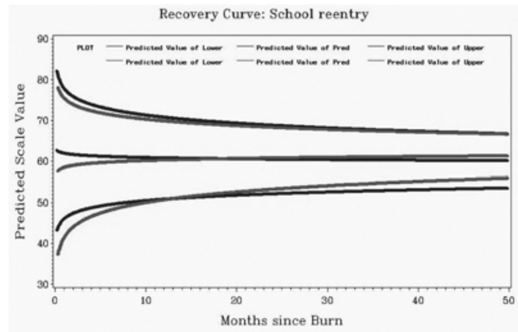

Fig. 10. School reentry

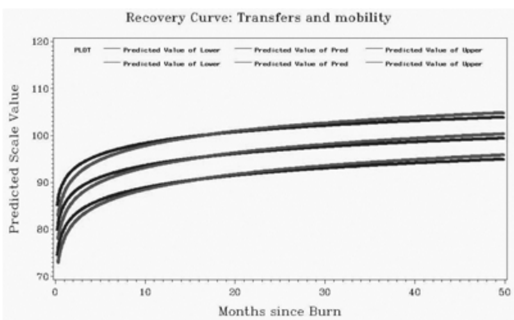

Fig. 9. Satisfaction

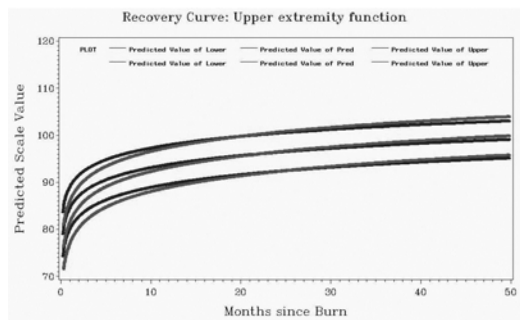

Fig. 10. School reentry

the parents short after the discharge. But later on, the parents' assessment is better than the children's.

CONCLUSION

In this paper, a comprehensive assessment method based on the generalized Gamma distribution model of the generalized linear mixed model is proposed, and the estimated parameters of the first hand data are used to construct the shrinkage estimation and the score test.

Through case analysis the researchers can see:

First and foremost, generalized linear mixed model of the generalized Gamma distribution can be estimated by the problem of the lack of observation and the repeated observation.

Second, in practice, the generalized linear mixed model of the generalized Gamma distribution model fits the data well. The researchers observe from the recovery curve that most of the twelve dimensions accurately reflect the positive effects of the medical care, but the comply (complain) decreases [d1] and the school (returning) score remains almost the same. These unexpected results do not have a direct explanation. The compliance may not reflect the effect of the medical care. Experience tells us that patients seem to following medical advices better in hospital than at home. On the other hand, children with burns, even though they have almost recovered, are not willing to come back to school. The patients may have personal or psychological issues dealing with the injuries, which causes them to hide from their peers. The model only contains data information in the visual display of information, the goal of building a model is to explain the model.

Third, taking into account the 3 layers of the loop nesting call for the implementation of the general linear mixed model module, the actual application of the algorithm has been very slow, in view of this, in the future need to be able to integrate the program to integrate.

Last but not least, the method proposed in this paper is also applicable to a comprehensive social analysis. By using this method, it can reflect the development and change of various conditions. It can be seen from the recovery curve that children can recover within 10 months after the treatment, but the social psychological barriers need to be sustained for 40 months.

The analysis presented in this research is not only applicable to medical and sociological studies, but can also be used towards their own research areas.

Future research can focus on the dependence structure of multi-variables. For example, the parents' assessment and the children's assessment have something in common. The dynamic assessment at different days may appear to have a strong correlation. This dependence structure may affect the parameter estimate and statistical inference.

ACKNOWLEDGEMENTS

The paper was financially supported by the National Social Science Foundation of China "Research on the Influence of China's Development and Strategic Environment affected by New International Energy Trend", 2015(152DA059), the Ministry of Education of the People's Republic of China, Philosophy Social Planning project, "Renewable energy industry's financing risk management and policy support system building-based on life cycle theory'', 2012 (12YJAZH056), the Energy Foundation (USA) projects, "Energy subsidies reform and the sustainable development of China's economy", 2011 (G-1111-15134), and the National Natural Science Foundation of China under grant no. 71303045 and "the Fundamental Research Funds for the Central Universities" in UIBE (No.15YQ09). This research was supported by the Zhejiang Provincial Natural Science Foundation of China under Grant No. LQ13G030011 for the study design and paper writing.

REFERENCES

Ahmad M, Adil N 2014. Compatibility and stability of polygeline (Haemaccel) with different drug products. *Pakistan Journal of Pharmaceutical Sciences*, 27(6): 2227–2235.

Antonio K, Beirlant J 2007. Actuarial statistics with generalized linear mixed models. *Insurance Mathematics and Economics*, 40(1): 58-76.

Baldi I, Berchialla P, Notaro V, Barone-Monfrin S, Bassi F, Gregori D 2010. Empirical assessment of gamma generalized linear mixed models for outcomes in oral health research. *Advances and Applications in Statistical Sciences*, 3(1): 43-64.

Breslow NE, Clayton DG 1993. Approximate inference in generalized linear mixed models. *JASA*, 88(3): 430-446.

Carroll RJ, Ruppert D 1988. *Transformation and Weighting in Regression*. London: Chapman and Hall.

Chen HC, Wehrly TE 2016. Approximate uniform shrinkage prior for a multivariate generalized linear mixed model. *Journal of Multivariate Analysis*, 145: 148–161.

Chen YI, Huang CS 2014. New approach to assess bioequivalence parameters using generalized gamma mixed-effect model (model-based asymptotic bioequivalence test). *Statistics in Medicine*, 33(5): 786-797.

Clayton DG, Cuzick J 1985. Multivariate generalizations of the Proportional Hazards Model (with discussion). *Journal of the Royal Statistical Society*, 148: 82-117.

Diaz FJ 2016. Measuring the individual benefit of a medical or behavioral treatment using generalized linear mixed-effects models. *Statistics in Medicine*, 35(23): 4077-4092.

Drikvandi R, Verbeke G, Molenberghs G 2016. Diagnosing misspecification of the random-effects distribution in mixed models. *Biometrics, Journal of The International Biometric Society*. DOI: 10.1111/biom.12551

Goldstein C 1991. *Essentials of Modern Literary Tibetan.* Berkeley: University of California Press.

Gurka MJ, Edwards LJ, Muller KE, Kupper LL 2006. Extending the Box-Cox Transformation to the Linear Mixed Model. *Journal of the Royal Statistical Society, Series A: Statistics in Society,* 169(2): 273-288.

Hougaard P 1986. A class of multivariate failure time distributions. *Biometrika*, 73: 671-678.

Hoyer A, Kuss O 2016. Meta-analysis for the comparison of two diagnostic tests to a common gold standard: A generalized linear mixed model approach. *Statistical Methods in Medical Research*.

Jiang J 2007. *Linear and Generalized Linear Mixed Models and Their Applications*. Berlin, Germany: Springer.

Keiding N, Andersen PK, Klein JP 1997. The role of frailty models and accelerated failure time models in describing heterogeneity due to omitted covariates. *Statist Med*, 16: 215-224.

Kwong GPS, Hutton JL 2003. Choice of parametric models in survival analysis: Applications to monotherapy for epilepsy and cerebral palsy. *Appl Statist*, 52: 153-168.

Lawless JF 2003. *Statistical Models and Methods for Lifetime Data*. 2nd Edition. New Jersey, US: John Wiley and Sons, Inc.

McCullagh P, Nelder JA 1989. *Generalized Linear Models*. 2nd Edition. London: Chapman and Hall.

Mcmahon C 2011. Generalized linear mixed model and calibration for gamma random variables: Application to asbestos fibers. *Dissertations and Theses - Gradworks*, 8(2): 225-236.

Mohani N, Ahmad M, Mehjabeen, Jahan N 2014. Evaluation of phytoconstituents of three plants Acorus calamus linn. Artemisia absinthium Linn and Bergenia himalaica boriss by FTIR spectroscopic analysis. *Pakistan Journal of Pharmaceutical Sciences*, 27(6): 2251–2255.

Nelder JA, Wedderburn RWM 1972. Generalized Linear Models. *Journal of the Royal Statistical Society*, 135(3): 370-384.

O'Hara RB 2016. How to make models add up- a primer on glmms. *Annales Zoologici Fennici*, 46(2): 124-137.

Verbeke G, Molenberghs G 2013. The gradient function as an exploratory goodness-of-fit assessment of the random-effects distribution in mixed models. *Biostatistics*, 14(3): 477-490.

Villemereuil PD, Schielzeth H, Nakagawa S, Morrissey M 2016. General methods for evolutionary quantitative genetic inference from generalized mixed models. *Genetics*, 204(3): 1281-1294.

Vonesh EF, Schaubel DE, Hao W, Collins AJ 2000. Statistical methods for comparing mortality among ESRD patients: Examples of regional/international variations. *Kidney Int*, 57: 19-27.

Wolfinger R, O'Connell M 1993. Generalized Linear Mixed Models: A pseudo-likelihood approach. *J Statistics, Computation Simulation*, 48(3): 233-243.

Xie YT, Yang J 2010. Generalized Linear Mixed Models based on generalized gamma distribution. *Statistics Research*, 27: 75 - 80.

Xie YT, Wang W, Tan YP, Yang J 2012. Credibility analysis based on generalized linear mixed models. *Statistics and Information Forum, Beijing, China*, 27(10): 3-8.

Xie YT, Yang J 2015. Shrinkage estimate for Generalized Linear Mixed Models based on Generalized Gamma Distribution. *Statistics and Decision, Beijing, China*, (10): 37-40.

Zeger SL, Liang KY, Albert PS 1988. Models for longitudinal data: A generalized estimating equation approach. *Biometrics*, 44: 1049-1060.

Paper received for publication on April 2016
Paper accepted for publication on February 2017